Section 2

Mathematics and Science

Reading Preview

Key Concepts

- What math skills do scientists use in collecting data and making measurements?
- What math skills help scientists analyze their data?

Key Terms

- estimate
- accuracy
- precision
- significant figures
- percent error
- mean
- median
- mode

Target Reading Skill

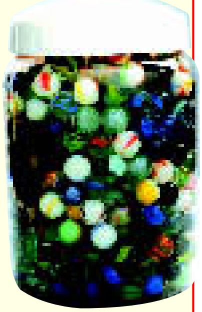
Asking Questions Before you read, preview the red headings. In a graphic organizer like the one below, ask a *what, how,* or *why* question for each heading. As you read, write the answers to your questions.

Mathematics and Science

Question	Answer
What does estimation have to do with science?	Scientists use estimation . . .

Lab zone Discover Activity

How Many Marbles Are There?



1. Your teacher will give you a jar full of marbles.
2. With a partner, come up with a way to determine the number of marbles in the jar without actually counting them.
3. Use your method to determine the number of marbles. Write down your answer.
4. Compare the method you used to that of another group.

Think It Over

Predicting Which method do you think led to a more accurate answer? Why?

Here's a riddle for you. What do the following things have in common: microscopes, telescopes, thermometers, balances, and mathematics? Do you give up? The answer is that they are all tools that scientists use.

Does it surprise you that mathematics is included in this list? You probably think of mathematics as something that is separate from science. But it is not. In fact, mathematics is sometimes called the "language of science."

Mathematics is essential for asking and answering questions about the natural world. From making measurements to collecting and analyzing data, scientists use math every day. This section focuses on some important math skills you will use in science class.

Estimation

Have you ever been on stage and wondered how many people there were in the audience? Maybe you counted the number of people in one row and multiplied by the number of rows. This would be one way to arrive at an estimate. An **estimate** is an approximation of a number based on reasonable assumptions. Estimating is not the same as guessing because an estimate is based on known information.

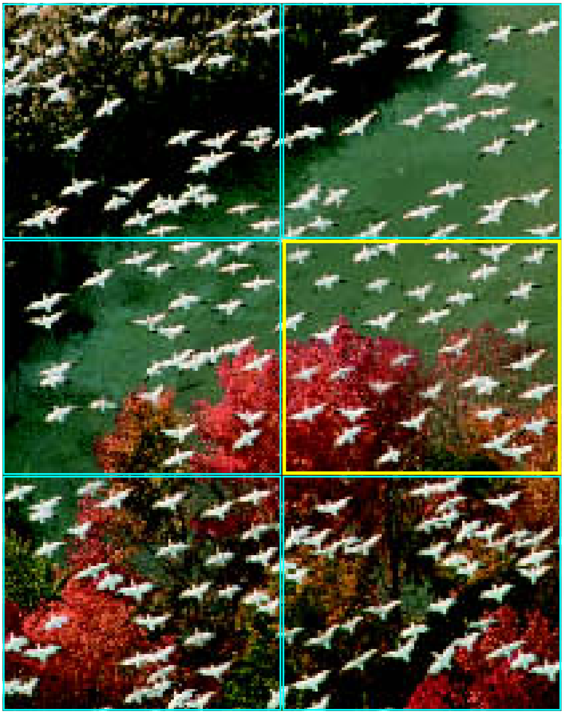
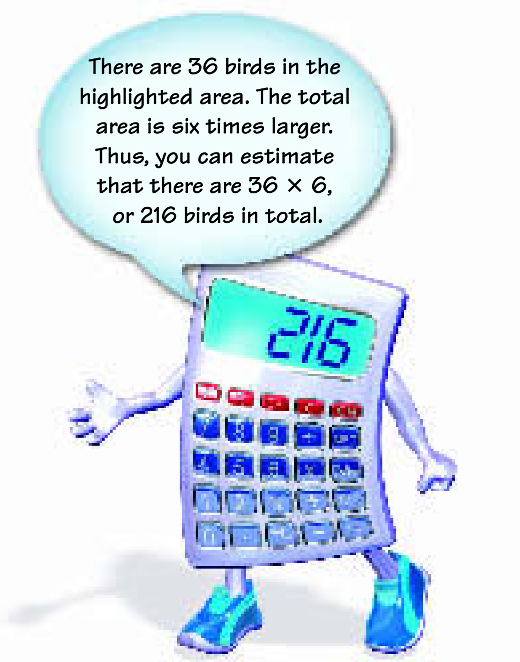


FIGURE 10
Estimation
Estimation is an important math skill that scientists use in their work. Estimating is a quick way to determine the large number of birds in this photo.



Scientists must sometimes rely on estimates when they cannot obtain exact numbers. Astronomers, for example, can't actually measure the distance between stars. Park rangers can't count the number of trees in large forests. Instead, scientists find ways to make reasonable estimates. An astronomer's estimate might be based on indirect measurements, calculations, and models. A park ranger's estimate might be based on a sample. The ranger could count the trees in a small area and then multiply to estimate the number in the entire forest.

Reading Checkpoint **What are estimates based on?**

Math Skills

Area

The area of a surface is the amount of space it covers. To find the area, multiply its length by its width. Remember to multiply the units as well.

Area = Length × Width

Suppose the area highlighted in Figure 10 measures 12.0 m by 11.0 m.

$$\text{Area} = 12.0\ \text{m} \times 11.0\ \text{m}$$
$$= 132\ \text{m}^2$$

Practice Problems Calculate the area of the following.

1. A room 4.0 m long and 3.0 m wide
2. A ticket stub 5.1 mm long and 2.62 mm wide

Accuracy and Precision

Suppose you were to meet a friend at 4:00 P.M. Your friend arrives at 4:15 and says, "But it's 4:00 according to all the clocks in my house." The problem is that your friend's clocks do not show the accurate, or correct, time. **Accuracy** refers to how close a measurement is to the true or accepted value. An accurate clock would read 4:00P.M. However, if all of your friend's clocks are always 15 minutes late, they can be said to be precise. **Precision** refers to how close a group of measurements are to each other.

Accuracy Versus Precision As you can see from the clock example, accuracy and precision do not mean the same thing. To understand the difference, think about a game of darts. As Figure 11 shows, accurate throws land close to the bull's-eye. Precise throws, on the other hand, land close to one another.

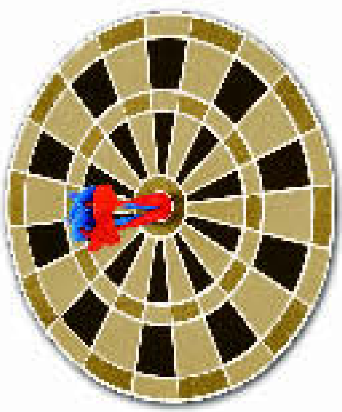
Accuracy and Precision in Measurements **Both accuracy and precision are important when you make measurements.** For example, suppose your younger sister wants to find out how tall she has grown. When you measure her height, the measurement needs to be accurate, or close to her true height. The measurement also needs to be precise. This means that if you measured her height several times, you would get the same measurement again and again.

How can you be sure that a measurement you make is both accurate and precise? First, you need to use a high-quality measurement tool. Next, you need to make your measurement carefully. Finally, you need to repeat the measurement a few times. If you follow these steps and get the same measurement each time, then you can feel confident that your measurement is reliable.

Reading Checkpoint **What does an accurate measurement mean?**

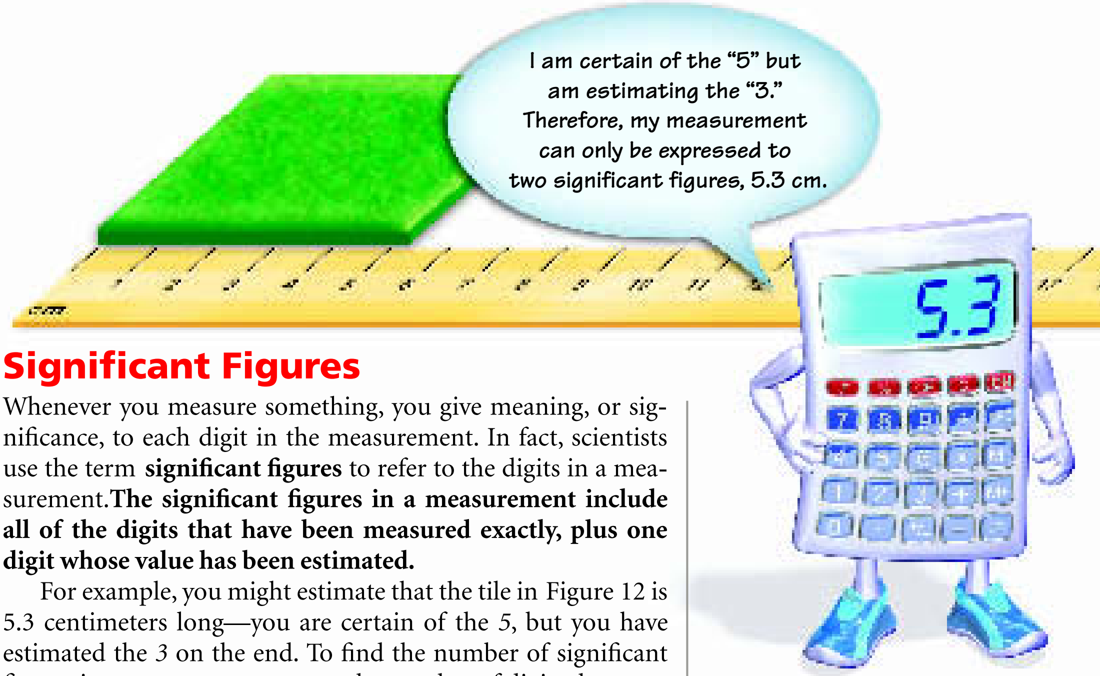
▲ Neither Precise nor Accurate

▲ Precise but Not Accurate



▲ Both Precise and Accurate

FIGURE 11
Accuracy and Precision
In a game of darts, it's easy to see the difference between accurate throws and precise throws. In order to hit the bull's-eye consistently, you need both accuracy and precision!



Significant Figures

Whenever you measure something, you give meaning, or significance, to each digit in the measurement. In fact, scientists use the term **significant figures** to refer to the digits in a measurement. **The significant figures in a measurement include all of the digits that have been measured exactly, plus one digit whose value has been estimated.**

For example, you might estimate that the tile in Figure 12 is 5.3 centimeters long—you are certain of the *5*, but you have estimated the *3* on the end. To find the number of significant figures in a measurement, count the number of digits that were accurately measured, plus the one estimated digit. Therefore, a measurement of 5.3 centimeters has two significant figures.

Adding or Subtracting Measurements When you add or subtract measurements, the answer can only have as many figures after the decimal point as the measurement with the fewest figures after the decimal. For example, suppose you add a tile that is 5.3 centimeters long to a row of tiles that is 21.94 centimeters long.

5.3 cm (1 significant figure after decimal)
+ 21.94 cm (2 significant figures after decimal)
27.24 cm = 27.2 cm (1 significant figure after decimal)

The answer, "27.2 centimeters," has only one significant figure after the decimal point because the measurement 5.3 centimeters has only one figure after the decimal point.

Why is it incorrect to express your answer as 27.24 centimeters? The reason is that your answer would appear to be more accurate than it really is. Remember that when you first measured the tile, you estimated the "3" in the tenths place. When you add that number to another, the number in the tenths place in the sum is still only an estimate. If you expressed your answer as "27.24" centimeters, the "2" in the tenths place would appear to be an exact measurement, rather than an estimate.

FIGURE 12
Significant Figures
A measurement should contain only those numbers that are significant—all of the digits that have been measured exactly plus one you have estimated.
Measuring *Why can you only report the length of the tile to two significant figures?*

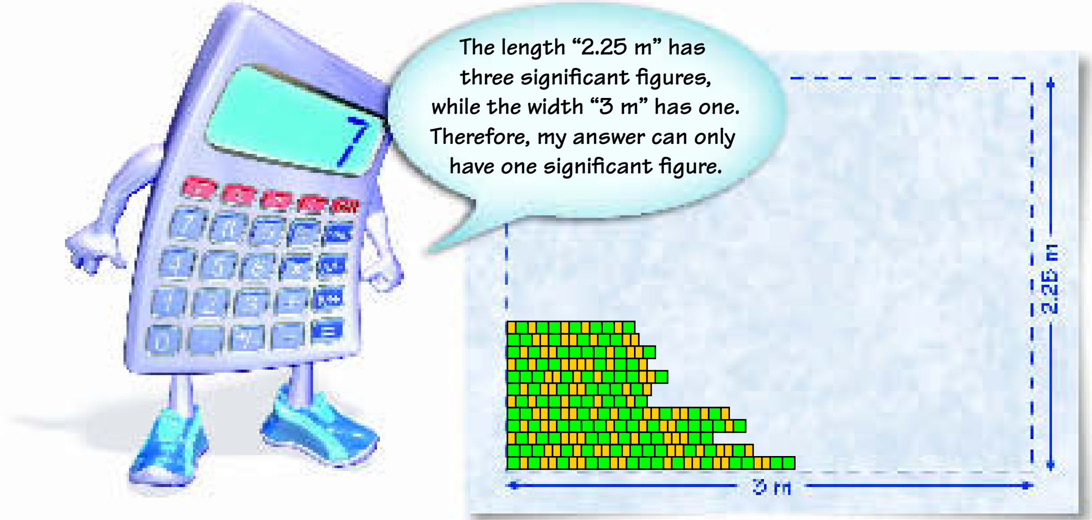


FIGURE 13
Multiplying Measurements
When you multiply measurements, your answer can only have the same number of significant figures as the measurement with the fewest significant figures.

Multiplying or Dividing Measurements You need to follow a slightly different rule when you multiply or divide measurements. When multiplying or dividing, the answer can only have the same number of significant figures as the measurement with the fewest significant figures.

Suppose you need to tile a space that measures 2.25 meters by 3 meters. The area of the space would be calculated as follows:

$$
\begin{array}{rl}
2.25\text{ m} & \text{(3 significant figures)} \\
\times\ 3\text{ m} & \text{(1 significant figure)} \\
\hline
6.75\text{ m}^2 = 7\text{ m}^2 & \text{(1 significant figure)}
\end{array}
$$

The answer has one significant figure because the least precise measurement (3 meters) has one significant figure.

Reading Checkpoint **What is the rule for multiplying or dividing measurements?**

Percent Error

"Today, class, your job is to determine the density of this metal." With those words, your science teacher hands you a small piece of a shiny metal. You get to work, carefully measuring its mass and volume. When you divide the mass by the volume, you arrive at a density of 9.37 g/cm^3. "That's pretty close," says your teacher, "but now you need to calculate your percent error. The correct value for the density of this metal is 8.92 g/cm^3."

Lab zone Try This Activity

For Good Measure

1. Estimate the following measurements without using any measurement tools. Be sure to use the correct units.
   a. the length of your desk
   b. the mass of a penny
   c. the volume of a shoe box
2. Measure each item above. Be sure to express each measurement to the correct number of significant figures.

Measuring How close were your estimates to the actual measurements?

Percent error calculations are used to determine how accurate, or close to the true value, an experimental value really is. To calculate **percent error,** use the following formula:

$$\text{Percent error} = \frac{\text{Difference between experimental value and true value}}{\text{True value}} \times 100\%$$

A low percent error means that the result you obtained was very accurate. A high percent error means that your result was not very accurate. You may not have made your measurements carefully enough or your measurement tool may have been of poor quality.

Math Sample Problem

Percent Error

You calculate the density of an object to be 9.37 g/cm^3. The density of the object is actually 8.92 g/cm^3. Calculate your percent error.

1 Read and Understand

What information are you given?

Experimental value = 9.37 g/cm^3

True value = 8.92 g/cm^3

2 Plan and Solve

What quantity are you trying to calculate?

Percent error = ■

What formula contains the given quantities and the unknown quantity?

$$\text{Percent Error} = \frac{\text{Difference between experimental value and true value}}{\text{True value}} \times 100\%$$

Perform the calculation.

$$\text{Percent error} = \frac{9.37\ \text{g/cm}^3 - 8.92\ \text{g/cm}^3}{8.92\ \text{g/cm}^3} \times 100\%$$

Percent error = **5.04%**

3 Look Back and Check

Does your answer make sense?

The answer tells you that your percent error is about 5%. This answer makes sense because the experimental value and the true value were close to each other.

Math Practice

Tanya measured the mass of an object to be 187 g. Sam measured the object's mass to be 145 g. The object's actual mass was 170 g.

1. What is Tanya's percent error?
2. What is Sam's percent error?

FIGURE 14
Finding Mean, Median, and Mode
A green sea turtle can lay dozens of eggs at a time. The average number of eggs per nest can be expressed as a mean, median, or mode.

Mean, Median, and Mode

Walking along a beach one summer night, you spot a green sea turtle. The turtle has laid its eggs in the warm sand. How many eggs does a green sea turtle lay? Suppose you count the eggs and find that there are 107. Do all green sea turtles lay 107 eggs? To find out, you would have to study many more turtle nests.

Figure 14 shows data from a survey of green sea turtle nests on the beach. Notice that the number of eggs ranges from 94 to 110. How can you use this data to find the "average" number of eggs? **There are several ways to determine an "average." They include the mean, median, and mode.**

Mean One type of average is called the mean. The **mean,** or numerical average, is calculated by adding up all of the numbers and then dividing by the total number of items in the list.

$$\text{Mean} = \frac{\text{Sum of values}}{\text{Total number of values}}$$

Median Sometimes it may be more useful to know the **median,** or the middle number in a set of data. To find the median, place all the numbers in order from smallest to largest. If the ordered list has an odd number of entries, the median is the middle entry in the list. If a list has an even number of entries, you can find the median by adding the two middle numbers together and dividing by two.

Mode A third way to represent an average is called the mode. The **mode** is the number that appears most often in a list of numbers. The mode is particularly useful when a list contains many numbers that are the same.

Reading Checkpoint **What is the median number in a list that has an odd number of entries?**

To Find the Mean

Add the numbers together and divide by the total number of items on the list.

Nest	Number of Eggs
A	110
B	102
C	94
D	110
E	107
F	110
G	109
Total	742

$$\text{Mean} = \frac{742 \text{ eggs}}{7} = 106 \text{ eggs}$$

To Find the Median

Place all the numbers in order from smallest to largest. The median is the middle entry.

94 102 107 (109) 110 110 110

Median = 109 eggs

To Find the Mode

Place all the numbers in order from smallest to largest. The mode is the number that appears most often.

94 102 107 109 (110 110 110)

Mode = 110 eggs

Section 2 Assessment

Target Reading Skill **Asking Questions** Work with a partner to check the answers about the section headings in your graphic organizer.

Reviewing Key Concepts

1. a. **Identifying** What math skill do scientists rely on when they cannot obtain exact numbers?
   b. **Explaining** Why is it important to obtain measurements that are both accurate and precise?
   c. **Interpreting Data** A friend measures the length of her room to be 3.7 meters. How many digits can you be certain of? Explain.
2. a. **Listing** What are three ways of calculating an "average"?
   b. **Problem Solving** Use all three ways to determine a student's "average" grade on eight exams: 88, 100, 92, 74, 90, 90, 84, 94.
   c. **Calculating** Suppose the student determined that his mean grade was 93. Calculate his percent error.

Math Practice

1. **Area** To win a prize at a fair, you must throw a coin into a space that is 7.0 cm long and 4.0 cm wide. What is the area of the space you are aiming for?
2. **Percent Error** You measured the cafeteria line to be 10.5 m long. The line is actually 6.2 m long. Calculate your percent error.